TAUP 2513-98

The Role of Inert Objects in Quantum Mechanical Phase

E. Comay

School of Physics and Astronomy

Raymond and Beverly Sackler Faculty

of Exact Sciences

Tel Aviv University

Tel Aviv 69978

Israel

Email: eli@tauphy.tau.ac.il

PACS No: 03.65.Bz

Abstract:

Quantum mechanical foundations of the polarized neutron phase shift experiment are discussed. The fact that the neutron retains its ground state throughout the experiment is shown to be crucial for the phase shift obtained.

Experimental data of the interaction of the neutron's magnetic moment with a magnetized material have two aspects: they provide information on the structure of specific materials and help us understand physical properties of the neutron. Thus, neutron scattering off a magnetized material was used for deciding that the neutron's magnetic moment is analogous to that of a current loop[1-3]. A new aspect of this issue is discussed in this work.

Very sensitive results of neutron dynamics are derived from an analysis of its interference pattern. Consider the nonrelativistic electromagnetic Lagrangian[4] augmented by the interaction of the neutron's intrinsic magnetic moment $\boldsymbol{\mu}$ with the external magnetic field $\boldsymbol{B}$ [5]

$$L = \frac{1}{2}mv^2 + e\boldsymbol{v}\cdot\boldsymbol{A} - eV + \boldsymbol{\mu}\cdot\boldsymbol{B}. \tag{1}$$

In this work expressions are written in units where $c = 1$. Obviously, in the case of a neutron, the electric charge e vanishes and we are left with the first and the last terms of (1). Hence, the action[4] derived from (1) depends on the external magnetic field, and one obtains

$$dS = (\frac{1}{2}mv^2 + \boldsymbol{\mu}\cdot\boldsymbol{B})dt. \tag{2}$$

This expression shows how the magnetic field affects the action, and thereby the neutron's phase and its interference pattern. Recent experiments utilize this relation between interference pattern and the Lagrangian (1)[6,7] for understanding the dynamics of the system. The present work analyses results of [7] and shows why quantum mechanical properties of the neutron distinguish it from an ordinary classical current loop.

In the experiment reported in [7], polarized neutrons drift through an external time dependent magnetic field. The neutron's velocity and spin as well as the external magnetic field are parallel to each other. At the neutron's location, the magnetic field is practically uniform. Hence, neither force nor torque are exerted on the neutron. The experiment confirms that, in spite of these facts and due to the action (2), the neutron is affected by the magnetic field which induces a phase shift that modifies the neutron's interference pattern.

Evidently, the neutron behaves in the experiment mentioned above like an inert object. Indeed, as a quantum mechanical system, the neutron is not excited to higher baryonic states whose energy is several hundreds Mev above the ground state, by its very small interaction with the external magnetic field. It is shown in this work that this property of the neutron is crucial for the magnetically dependent phase shift of (2).

Let us examine a hypothetical experiment which is similar to the one described in [7]. All parts of this experiment are like those of [7], except the neutron which is replaced by a "classical neutron" whose structure is described below.

Consider a ring made of an insulating material. This material takes the shape of a thin circular pipe containing a positively charge fluid. The insulating material is charged uniformly with negative charge so that the device looks like an electrically neutral object. The charge to mass ratio of the fluid is very small. This fluid rotates frictionlessly in a clockwise direction. Let a denote the ring's radius and I is the electric current associated with the fluid's motion. The magnetic field of the device is like that of a tiny magnetic dipole[8]

$$\boldsymbol{\mu} = \pi a^2 I \boldsymbol{k} \tag{3}$$

where $\boldsymbol{k}$ denotes a unit vector in the z-direction. This hypothetical experiment is carried out in conditions where the nonrelativistic limit holds. The rather small ring is a macroscopic object and the motion of the charged fluid can be treated classically.

The system's Lagrangian is the ordinary Lagrangian of an electromagnetic system[4], namely (1) without its last term

$$L = \frac{1}{2}mv^2 + e\boldsymbol{v} \cdot \boldsymbol{A} - eV. \tag{4}$$

Here the electric potential V vanishes. Hence, for evaluating (4) we have to find the values of the mechanical part and that of the second one which is associated with the magnetic interaction. The mechanical part of the Lagrangian describing the present experiment consists of two terms. One term pertains to the motion of the device in the z-direction and the second one is associated with the rotation of the fluid. Hence, the required Lagrangian is

$$L = \frac{1}{2}Mv_z^2 + \frac{1}{2}m_f v_\perp^2 + \int \boldsymbol{j} \cdot \boldsymbol{A} d^3r \tag{5}$$

where M and m_f denote the mass of the entire device and of the rotating fluid, respectively and $v_\perp$ is the fluid's velocity in the (x, y) plane. The last term of (5) is the continuum analog of the single particle expression $e\boldsymbol{v} \cdot \boldsymbol{A}$ (see [8], p. 596).

Let us evaluate the last term of (5). Introducing the electric current I, one applies Stokes theorem, the spatial uniformity of the magnetic field $\boldsymbol{B}$ and relation (3) and finds

$$\begin{aligned} \int \boldsymbol{j} \cdot \boldsymbol{A} d^3r &= I \oint_L \boldsymbol{A} \cdot d\boldsymbol{l} \\ &= I \int_S (curl \boldsymbol{A}) \cdot d\boldsymbol{s} \\ &= \pi a^2 I B \\ &= \boldsymbol{\mu} \cdot \boldsymbol{B}. \end{aligned} \tag{6}$$

Here the integral subscript L denotes the closed path along the ring and S is the ring's area. It follows that in (5), the term $\boldsymbol{j} \cdot \boldsymbol{A}$ of the classical device discussed here is analogous to the last term of (1).

Unlike the neutron case, where its internal structure does not change while the external magnetic field is turned on, the kinetic energy of the rotating fluid depends on the external magnetic field. Since the charge to mass ratio of the rotating fluid is very small, the current I is regarded in the following calculation as a constant. The power transmitted to the rotating charged fluid is

$$\begin{aligned} P &= I \oint_L \boldsymbol{E} \cdot d\boldsymbol{l} \\ &= I \int_S (curl \boldsymbol{E}) \cdot d\boldsymbol{s} \\ &= -I\pi a^2 \frac{\partial B}{\partial t} \end{aligned} \tag{7}$$

where the spatial uniformity of $\boldsymbol{B}$ is used. Applying the notation of (5), let $v_\perp(T_0)$ denote the velocity of the charged fluid at T_0 before the external magnetic field is turned on. It follows that, at an instant t, this part of the kinetic energy of the fluid is

$$\begin{aligned} \frac{1}{2} m v_\perp^2(t) &= \frac{1}{2} m v_\perp^2(T_0) - I\pi a^2 \int \frac{\partial B}{\partial t} dt \\ &= \frac{1}{2} m v_\perp^2(T_0) - I\pi a^2 B. \\ &= \frac{1}{2} m v_\perp^2(T_0) - \boldsymbol{\mu} \cdot \boldsymbol{B}. \end{aligned} \tag{8}$$

Substituting (6) and (8) into (5), one finds that in the classical analog of the neutron experiment, the value of the Lagrangian

$$L(t) = \frac{1}{2} M v_z^2 + \frac{1}{2} m_f v_\perp^2(T_0) = L(T_0) \tag{9}$$

is independent of the external magnetic field. The same is true for the corresponding action.

The results of this work emphasize the quantum mechanical meaning of the polarized neutron interference experiment[7]. As stated in [7], the neutron is free of classical effects like force and torque. In spite of this, its quantum properties vary due to the external magnetic field. The discussion carried out above points out another quantum mechanical aspect of the experiment. The neutron's spin and its associated magnetic moment are properties of a quantum mechanical system whose state may vary only in quantum leaps. The excited baryonic states of the neutron are very high, so that in experiment [7], the neutron is always in its ground state and behaves as an inert object. As such, there exists no analog to the variation of the self energy of the rotating fluid (8). For this reason, the Lagrangian of the neutron experiment [7] and the corresponding action *depend* on the external magnetic field whereas in the analogous classical experiment discussed above they are *independent* of it. It can be concluded that a comparison of the two experiments, namely the actual experiment reported in [7] and the hypothetical one which uses a classical current loop, demonstrates another quantum mechanical aspect of the neutron experiment [7].

References:

[1] C. G. Shull, E. O. Wollan and W. A. Strauser, Phys. Rev. **81** (1951) 483.

[2] F. Mezei, Physica **137B-C** (1986) 295.

[2] F. Mezei, Physica **151B-C** (1988) 74.

[4] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Pergamon, Oxford, 1975). p. 46.

[5] J. M. Blatt and V. F. Weisslopf, *Theoretical Nuclear Physics* (Wiley, New York, 1952) p. 32.

[6] J. Summhammer et al., Phys. Rev. Lett. **75** (1995) 3206.

[7] W. -T Lee, O. Motrunich, B. E. Allman and S. A. Werner, Phys. Rev. Lett. **80** (1998) 3165.

[8] J. D. Jackson, *Classical Electrodynamics,* second edition (John Wiley, New York, 1975). pp. 178, 179.